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UNIVERSAL KRIGING IN MULTIPARAMETER TRANSDUCER CALIBRATION

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Abstract

The paper presents the universal kriging method applied in calibration of multiparameter transducers. If a measured transducer characteristic is not within an assumed error margin, it is necessary to perform calibration to establish its individual transfer function. The universal kriging method may be then applied in order to evade repeating the measurements for every considered transducer, thus saving significant amounts of time.

Keywords: correction of transducer transfer function, A/D converter.

1. Introduction

The transducer manufacturing process can influence its key metrological parameters. Therefore performing calibration of each transducer is a necessity. In case of one-parameter transducers it is relatively easy even in those with a non-linear transfer function. However, performing calibration for a multi-parameter transducer is a far more complicated problem. Commonly used methods apply to narrow ranges of transducers, an example of which can be found in [3] – a review of calibrating methods for spectroscopy. A calibration method incorporating multiple influencing parameters with the use of the Gauss approximating function can be found in [5]. Another method for gas flow transducer calibration is described in [8]. Also, some aspects of complex measurements are discussed in [14].

The paper presents a calibration method for multiparameter transducers. The method is based on the assumption that for at least one transducer from the batch the transfer function has already been established. A correction function for the others is found using the universal kriging method.

2. Kriging method

In universal kriging it is assumed that the unknown function values are expressed by a combination of two components: a deterministic, where value depends on the position and a stochastic, with a constant average and fulfilling the condition of second-degree stationarity:

$$F(s) = g(s) + Z(s), \tag{1}$$

where:
- $g(s)$ – deterministic component;
- $Z(s)$ – stochastic component.

From a formal point of view, universal kriging is, similarly to pointwise kriging, considered to be a random function $F(s)$ defined in a space S, where $s(x, y,\dots,z)$ are the coordinates in this space. The discrete values of $F(s)$ are known in s_i $(1 \le i \le n)$ nodes, where n is the number of nodes in space S. It is assumed that only the points in the vicinity of a point

s_p influence the F(s) value. The area containing those adjoining nodes is called the influence or interpolation area.

Various function types are used to describe the deterministic component. Polynomials with monomials, orthogonal polynomials, sine functions as basis are widely used. In case of a transducer mathematical model, its function is either strictly ascending or descending, therefore, with some exceptions, polynomial functions are preferred as the deterministic component. Hence the deterministic component can be expressed as a linear combination of m known functions with weight coefficients of a_k:

$$g(s) = \sum_{k=0}^{m} a_k f_k(s). \tag{2}$$

The assumption that the average value of the stochastic component equals zero is acceptable and does not change the universality of considerations . It can be achieved by normalizing the measurement data. Then we will have:

$$E[Z(s)] = 0. \tag{3}$$

The average value of the entire function will be expressed by:

$$E\left[F(s)\right] = \sum_{k=0}^{m} a_k f_k(s) \tag{4}$$

and the covariance of the stochastic component will be as follows:

$$E[(F(s_1) - g(s_1))(F(s_2) - g(s_2))] = E[Z(s_1)Z(s_2)] = Cov(s_1 - s_2). \tag{5}$$

Generally, as in simple kriging [2, 19] the expected value of the F(s) function in s_p can be derived from the expression:

$$\hat{F}(s_p) = \sum_{i=1}^{n} \omega_i F(s_i), \tag{6}$$

where:

- $F(s_i)$ – function values in nodes s_i (i = 1, 2, …, n);
- n – number of nodes;
- ω_I – weights corresponding to nodes.

The weights ω_I are derived from the process of finding an unbiased estimator with the minimal error between the real and expected value. The unbiased conditions are when:

$$E\left[\hat{F}(s_p)\right] = E\left[F(s_p)\right]. \tag{7}$$

Taking into account the above expressions, one can write:

$$g(s_p) = \sum_{i=1}^{n} \omega_i g(s_i) \tag{8}$$

and inserting (2) into (8) we get:

$$\sum_{k=0}^{m} a_k f_k(s_p) = \sum_{i=1}^{n} \omega_i f_k(s_i). \tag{9}$$

To satisfy equation (9) we have a sequence of conditions:

$$f_k(s_p) = \sum_{i=1}^{n} \omega_i f_k(s_i) \qquad k = 0,\dots, m. \tag{10}$$

Basically, to find the ω_i weights, the following variance is minimized:

$$\sigma^2 = E\left[\hat{F}(s_p) - F(s_p)\right]^2. \tag{11}$$

Expanding (11) gives:

$$\sigma^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\omega_i\omega_j \mathrm{Cov}(s_i,s_j) - 2\sum_{i=1}^{n}\omega_i \mathrm{Cov}(s_i,s_p) + \mathrm{Var}(F(s_p)). \tag{12}$$

If the covariance with distance h between particular points s is inserted into expression (12), one can get:

$$\sigma^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\omega_i\omega_j \mathrm{Cov}(h_{ij}) - 2\sum_{i=1}^{n}\omega_i \mathrm{Cov}(h_i) + \mathrm{Var}(F(s_p)), \tag{13}$$

where:

- h_{ij} – Euclidean distance between points s_i, s_j;
- h_i – Euclidean distance between points s_i, s_p.

The minimization of expression (13) in relation to ω_i, is carried out by incorporation of Lagrange coefficients μ_r as it is in the case of ordinary kriging. It results in (k+n) linear equations with (k+n) unknowns. Basing on the analysis carried out in [15] the covariances in (13) can be substituted by $\gamma(h)$ semivariances. This results in the following set of equations:

$$\sum_{i=1}^{n}\omega_i\gamma(h_{ij}) + \sum_{k=0}^{m}\mu_k f_k(s_i) = \gamma(h_i) \quad i = 1,\dots, n \tag{14}$$

$$\sum_{j=1}^{n}\omega_j f_k(s_j) = f_k(s_p) \qquad k = 0, \dots, m \tag{15}$$

where, similarly like in ordinary kriging:

- $\gamma(h_{ij})$ – semivariance in points s_i and s_j.
- $\gamma(h_i)$ – semivariance in points s_i and s_p.
- μ_k – Lagrange coefficients.

The weight coefficients ω obtained as a solution to equations (14) and (15), are used in (6) - to predict the function value in selected point s_p as well as in (13) – to determine the variance. When random variables have a Gauss distribution, the points of prediction are in the following interval :

$$\left|\hat{F}(s_p) - 1.96\sigma_{sp}, \hat{F}(s_p) + 1.96\sigma_{sp}\right| \tag{16}$$

with a confidence level of 95%.

Assuming that $f_l = 1$, and the other functions f_2, … $f_m = 0$, we get a set of equations for ordinary kriging.

The set of equations (14) and (15) can be expressed in matrix syntax:

$$\mathbf{Gc = g}, \tag{17}$$

where:

$$\mathbf{G} = \begin{vmatrix} \gamma(0) & \dots & \gamma(h_{1n}) & f_0(s_1) & \dots & f_m(s_1) \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \gamma(h_{n1}) & \dots & \gamma(0) & f_0(s_n) & \dots & f_m(s_n) \\ f_0(s_1) & \dots & f_0(s_n) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ f_m(s_1) & \dots & f_m(s_n) & 0 & \dots & 0 \end{vmatrix}, \tag{18}$$

$$\mathbf{c} = [\omega_1 \quad \omega_2 \ldots \omega_n \quad \mu_1 \quad \mu_2 \ldots \mu_m]^T \tag{19}$$

or
$$c = \begin{vmatrix} \boldsymbol{\omega} \\ \boldsymbol{\mu} \end{vmatrix}, \tag{20}$$

where: $\boldsymbol{\omega} = [\omega_1 \quad \omega_2 \ldots \omega_n]^T$, $\boldsymbol{\mu} = [\mu_1 \quad \mu_2 \ldots \mu_m]^T$

$$\mathbf{g} = [\gamma(h_1) \quad \ldots \quad \gamma(h_n) \quad f_0(s_p) \quad \ldots \quad f_m(s_p)]^T \tag{21}$$

or
$$\mathbf{g} = \begin{vmatrix} \boldsymbol{\gamma}(h) \\ \mathbf{f}(s_p) \end{vmatrix}, \tag{22}$$

where: $\gamma(h) = [\gamma(h_1) \quad \gamma(h_2) \ldots \gamma(h_n)]^T$, $\mathbf{f}(s_p) = [f_0(s_p) \quad f_2(s_p) \ldots f_m(s_p)]^T$.

Expressions (18) to (20) can be expressed as:

$$\mathbf{G} = \begin{vmatrix} \boldsymbol{\Gamma} & \mathbf{F}(s) \\ \mathbf{F}(s)^T & \mathbf{0} \end{vmatrix}, \tag{23}$$

where: $\boldsymbol{\Gamma} = \begin{vmatrix} 0 & \gamma(h_{12}) & \gamma(h_{1n}) \\ \gamma(h_{21}) & 0 & \gamma(h_{2n}) \\ \gamma(h_{n1}) & \gamma(h_{n2}) & 0 \end{vmatrix}$.

$$\mathbf{F}(s) = \begin{vmatrix} f_0(s_1) & f_0(s_n) \\ f_m(s_1) & f_m(s_n) \end{vmatrix}. \tag{24}$$

Just as in case of ordinary kriging, in universal kriging some of the random function distribution semivariogram models [16, 18] can be applied for semivariance computing. The most often used models are Gauss, exponential, spatial and power function distribution. The model is selected basing on experimentally validated ones, alternatively suitable tests should be performed on measurement data in order to determine the proper model of the semivariogram.

The **c** vector of ω weights and Lagrange coefficients μ is derived from the expression:

$$\mathbf{c} = \mathbf{G}^{-1}\mathbf{g}. \tag{25}$$

After calculating (21) and determination of weights ω, the expected value in point s_p and the variance can be determined from the following:

$$\sigma^2(s_p) = \sum_{i=1}^{n} \omega_i \gamma(h_i) - \sum_{k=1}^{m} \mu_k f_k(s_p). \tag{26}$$

3. Experimental design

The universal kriging method was applied to calculate corrective polynomials for gas flow transducers [6, 7]. The transducer range of measurements was 0.04 - 0.8 m^3/h. The transducer contained a pneumatic resistance and the Q value was calculated from the dependence (27) basing on the pressure drop on the pneumatic resistor dP with regard to the temperature T_a and the absolute pressure P_a:

$$Q = \frac{-a_1 \frac{\eta(T_a)}{\eta(T_o)} + \sqrt{\left(a_1 \frac{\eta(T_a)}{\eta(T_o)}\right)^2 + 4a_2 \frac{P_a T_o}{P_o T_a}(dP - a_0)}}{2a_2 \frac{P_a T_o}{P_o T_a}}, \tag{27}$$

where:

- Q – value of flow [m^3/h];
- T_0 – reference temperature 273.15 [K];
- P_0 – reference absolute pressure 100 kPa;
- T_a – temperature of measurement [K];
- P_a – pressure of measurement [kPa];
- $\eta(T_0)$ – gas viscosity coefficient at reference temperature;
- $\eta(T_a)$ – gas viscosity coefficient at temperature of measurement;
- dP – pressure drop on the pneumatic resistor;
- a_i – coefficients.

The a_i coefficients in expression (27) were calculated for a particular transducer, Table1.

Table 1. Values of the coefficients a_i in expression (27).

a_2	a_1	a_0
286.44	68.654	2.631

Basing on measurement data, the dQ between the real Q_r and Q value was calculated for all others using the expression (27). Values of the coefficients dP, P_a, i T_a were taken from the range: dP {0 – 250 Pa}, P_a {100 – 120 kPa}, T_a {273.15 – 308.15 K}. Table 2 presents measurement values of the parameters, randomly determined. For dP, P_a and T_a values, as corrective function the expression (28) was chosen.

Table 2. Measured values of the parameters dp, P_a, T_a, and dQ.

No	P_a [kPa]	T_a [k]	dP [Pa]	dQ [m^3/h]	No	P_a [kPa]	T_a [k]	dP [Pa]	dQ [m^3/h]
1.	118.7	300.4	132.6	0.0252	10.	119.2	276.8	110.6	0.0198
2.	109.4	293.0	32.4	0.0064	11.	116.3	300.2	1.1	0.0021
3.	103.2	284.9	2.9	0.0028	12.	108.0	276.1	217.1	0.0525
4.	110.6	284.0	198.5	0.0425	13.	108.6	301.1	64.9	0.0103
5.	105.3	294.2	41.4	0.0079	14.	105.2	279.5	227.6	0.0538
6.	115.0	297.2	163.5	0.0323	15.	117.3	277.9	36.3	0.0068
7.	104.6	276.0	112.6	0.0259	16.	102.8	292.3	144.9	0.0235
8.	116.5	278.4	228.	0.0574	17.	107.0	294.9	213.2	0.0507
9.	101.5	308.0	134.5	0.0240	18.	101.5	287.2	128.3	0.0219

With a deterministic function the semivariance functions (18) were determined and the α_I coefficients α_I , β_i and γ_i for the other transducers were computed using expressions (23), (24) and (25).

$$\Delta Q = \sum_{i=0}^{2} \left(\alpha_i P_a + \beta_i T_a + \gamma_i P_a T_a\right) dP^i . \tag{28}$$

Values of the coefficients α_i, β_i and γ_i are presented in Table 3.

Table 3. Values of the coefficients α_I, β_i, γ_i in expression (28).

i	α_i	β_i	γ_i
0	3.251e-5	1.102e-5	-1.348e-7
1	1.437e-6	4.278e-7	-5.372e-9
2	5.635e-9	1.682e-9	-2.144e-11

The test of the volume flow meter show the errors of the measurements less then 0.8%.

The sensitivity of the above method for the measurement errors of the parameters Pa, Ta and dP was tested in a simulation manner. The values of the parameters were randomly changed in the range ± 1% and once more the function (28) was determined. Comparison of the values obtained from both functions show differences less then 0.6%. This indicates that for normal distribution of the measurement errors, they are averaged.

The most challenging problem was to find the corrective function (28). It was achieved using the method of successive approximations. However, after it was found at last, a set of 18 measurements distributed in the whole measurement space was enough to determine the α_i, β_i i γ_i coefficients. The corrective function format is the same for all the transducers.

The above example shows that the above method is very efficient. This results from the fact that the transfer function of the transducer is smooth and strictly ascending or strictly descending.

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